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# Dressed giant magnons on $\mathbb{CP}^3$

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ABSTRACT: A new example of AdS/CFT duality relating IIA string theory on  $AdS_4 \times \mathbb{CP}^3$ to  $\mathcal{N} = 6$  superconformal Chern-Simons theory has recently been provided by ABJM. By now a number of papers have considered particular giant magnon classical string solutions in the  $\mathbb{CP}^3$  background, corresponding to excitations in the spin chain picture of the dual field theory. In this paper we apply the  $\mathbb{CP}^3 = SU(4)/S(U(3) \times U(1))$  dressing method to the problem of constructing general classical string solutions describing various configurations of giant magnons. As a particular application we present a new giant magnon solution on  $\mathbb{CP}^3$ . Interestingly the dressed solution carries only a single SO(6) charge, in contrast with the dyonic magnons found in previous applications of the dressing method.

KEYWORDS: Long strings, AdS-CFT Correspondence

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## 1 Introduction

Motivated by the work of Bagger, Lambert and Gustavsson [1] on maximally superconformal field theories in three dimensions, Aharony, Bergman, Jafferis, and Maldacena (ABJM) constructed [2] an  $\mathcal{N} = 6$  superconformal Chern-Simons theory with  $U(N) \times U(N)$  gauge symmetry at levels (k, -k) that is believed to be dual to M-theory on  $\mathrm{AdS}_4 \times S^7/Z_k$  (see also [3]). ABJM further considered the  $N, k \to \infty$  limit keeping the 't Hooft coupling  $\lambda = N/k$  fixed and conjectured that in this limit the  $\mathcal{N} = 6$  field theory is dual to type IIA string theory on  $\mathrm{AdS}_4 \times \mathbb{CP}^3$ .

Given the important role that integrability has played in exploring the structure of  $\mathcal{N} = 4$  Yang-Mills theory and its dual, it is natural that this new example of AdS/CFT provides an arena for further studying aspects of integrability in gauge/string duality. The worldsheet theory for IIA strings on  $AdS_4 \times \mathbb{CP}^3$  has been constructed and its possible integrability explored in [4], while on the Chern-Simons side the anomalous dimensions of local operators are apparently encoded in integrable spin chain Hamiltonian [5]. An exact magnon S-matrix for this spin chain has been proposed in [6], numerous tests of these proposals have been carried out in [7], and aspects of Wilson loops have been studied in [8].

Hofman and Maldacena [9] identified the string theory dual of an elementary magnon in the spin chain description of  $\mathcal{N} = 4$  Yang-Mills theory as a particular classical open string configuration on an  $\mathbb{R} \times S^2$  subset of  $\mathrm{AdS}_5 \times S^5$ , called the giant magnon. The study of giant magnons and their BPS bound states [10] has provided a wealth of detailed information about AdS/CFT. Naturally therefore a number of papers [11–14] have explored in detail the properties of various giant magnon solutions relevant to the ABJM incarnation of  $\mathrm{AdS}_4/\mathrm{CFT}_3$ .

The dressing method of Zakharov and Mikhailov [15] provides an algorithm to directly construct solutions of classically integrable equations. This method has proven useful for the construction of various giant magnon solutions, including magnons on spheres [16–18] and on anti-de Sitter space [19]. An explicit solution describing the scattering of N giant magnons on  $\mathbb{R} \times S^3$  was also presented in [20], and their dynamics on  $S^2$  were studied in [21]. Since the equations of motion for a string on  $\mathbb{R} \times \mathbb{CP}^3$  are also classically integrable, these techniques can be employed here as well. In this paper we demonstrate the application of the dressing method for  $\mathrm{SU}(4)/S(\mathrm{U}(3) \times \mathrm{U}(1))$  coset model (due to Harnad et. al. [22]) to the problem of constructing  $\mathbb{CP}^3$  giant magnon solutions.

An important feature of the dressing method, which has been exploited for example in [16–18, 20], is that repeated application can be used to generate explicit classical string solutions describing the scattering of any number of giant magnons (or, when applicable, bound states thereof). In complex sigma models each application of the dressing method introduces two new non-trivial parameters into the solution, which in familiar cases [16, 17] specify the magnon's momentum p and the value of a second SO(6) charge  $J_2$  which it carries in addition to the charge J carried by the ground state in an orthogonal plane. Such dressed solutions are therefore dyonic giant magnons of the type introduced by Dorey [10]. However in the present application of the dressing method we find something of a surprise — the dressed solution given below in (4.6) indeed has two parameters, but it carries only a single SO(6) charge J, having  $J_2 = 0$ . The dressing method apparently finds a oneparameter family of solutions for any fixed value of the charges p and J, but no dyonic magnons. We do not present any multi-magnon solutions here, but the algebra involved is no more complicated than for the solutions studied in [17, 20].

Note added. The results in this note were obtained some time ago but were delayed due to the disappointment of not being able to find more general (two-charge) solutions. We were motivated to publish our results by the recent appearance of [14] which includes, in addition to a formula (7.15) for the new  $\mathbb{CP}^3$  magnon that is identical to our solution (4.6), a very thorough analysis demonstrating that the dressing method by itself cannot produce any such "dyonic" solutions. This conclusion has also been reached in [23]. Elementary dyonic giant magnon solutions on  $\mathbb{CP}^3$  have finally been obtained in [24] for a special value of p and in [25] generally. The solution (4.6) below is evidently a neutral composite of two such elementary magnons [25].

# 2 The $\mathbb{CP}^3$ model

The  $\mathbb{CP}^3$  model may be described by a complex four-component vector n with lagrangian density

$$2\sqrt{2\lambda}\mathcal{L} = -\partial_{\mu}n^{\dagger}\cdot\partial^{\mu}n + (n^{\dagger}\cdot\partial_{\mu}n)(\partial^{\mu}n^{\dagger}\cdot n) - \Lambda(n^{\dagger}\cdot n - 1)$$
(2.1)

where  $\lambda = N/k$  is the 't Hooft coupling. The Lagrange multiplier constrains the fields to lie on  $S^7 \in \mathbb{C}^4$ , while the local U(1) invariance of (2.1) allows us to identify  $n \sim e^{i\Lambda(x)}n$ , thereby reducing the configuration space to  $S^7/U(1) = \mathbb{CP}^3$ . The action possesses an SU(4) symmetry with Noether currents

$$J^{a}_{\mu} = 2\sqrt{2\lambda} \operatorname{Im}[(n^{\dagger} \cdot T^{a}\partial_{\mu}n) - (n^{\dagger} \cdot T^{a}n)(n^{\dagger} \cdot \partial_{\mu}n)], \qquad (2.2)$$

where  $T^a$  are generators of SU(4). The equations of motion (after eliminating the Lagrange multiplier) are

$$-\partial^2 n + (n^{\dagger} \cdot \partial^2 n)n + 2(n^{\dagger} \cdot \partial_{\mu} n)\partial^{\mu} n + 2(\partial_{\mu} n^{\dagger} \cdot n)(n^{\dagger} \cdot \partial^{\mu} n)n = 0.$$
(2.3)

To describe classical strings on  $\mathbb{R} \times \mathbb{CP}^3$  (with a trivial time coordinate), the equations of motion must be supplemented with the Virasoro constraints

$$(\partial_{+}n^{\dagger} \cdot \partial_{+}n) - (n^{\dagger} \cdot \partial_{+}n)(\partial_{+}n^{\dagger} \cdot n) = \frac{1}{4},$$
  

$$(\partial_{-}n^{\dagger} \cdot \partial_{-}n) - (n^{\dagger} \cdot \partial_{-}n)(\partial_{-}n^{\dagger} \cdot n) = \frac{1}{4},$$
(2.4)

where we have used light-cone coordinates  $x_+ = \frac{1}{2}(x-t)$ ,  $x_- = \frac{1}{2}(x+t)$  and the derivatives are with respect to those coordinates,  $\partial_+ = \partial_x - \partial_t$ ,  $\partial_- = \partial_x + \partial_t$ .

Several classes of solutions to the equation of motion (2.3) and the Virasoro constraints (2.4) may be obtained by embedding known giant magnon solutions that live on  $S^2$  or  $S^3$  into  $\mathbb{CP}^3$  (an extensive discussion of these embeddings has been given in [13]). As a first example, let  $(X_1, X_2, X_3)$  be coordinates satisfying  $X_1^2 + X_2^2 + X_3^2 = 1$ . An isometric embedding  $S^2 \to \mathbb{CP}^3$  is given by

$$n^{\mathrm{T}} = \frac{1}{\sqrt{2(1-X_3)}} \begin{pmatrix} X_1 + iX_2 & 1 - X_3 & 0 & 0 \end{pmatrix}.$$
 (2.5)

In this manner any solution  $X^i = (X_1, X_2, X_3)$  of string theory on  $\mathbb{R} \times S^2$ 

$$-\partial^2 X^i + (X \cdot \partial^2 X) X^i = 0,$$
  

$$\partial_+ X \cdot \partial_+ X = \partial_- X \cdot \partial_- X = 1,$$
(2.6)

lifts to a solution of (2.3) and (2.4). In general it may be necessary to rescale the worldsheet coordinates x, t in order to satisfy (2.4) with the normalization shown. Such a rescaling does not affect the equations of motion (2.3). Similarly we can consider what has been called the " $S^2 \times S^2$ " embedding in the literature. This is given by the map

$$n^{\mathrm{T}} = \frac{1}{2\sqrt{(1-X_3)}} \begin{pmatrix} X_1 + iX_2 & 1 - X_3 & X_1 + iX_2 & 1 - X_3 \end{pmatrix}$$
(2.7)

whose image inside  $\mathbb{CP}^3$  is actually [13] just a single  $S^2$  partially rotated into two orthogonal directions compared to (2.5).

In the case of magnons living on  $S^3$  we can parameterize the unit 3-sphere with embedding coordinates  $X^i = (X_1, X_2, X_3, X_4)$ . Given any such solution  $X^i$  describing a classical string on  $\mathbb{R} \times S^3$  there are two possible natural embeddings into a solution of the  $\mathbb{CP}^3$ equations, given alternately by

$$n^{\mathrm{T}} = \begin{pmatrix} X_1 & X_2 & X_3 & X_4 \end{pmatrix} \tag{2.8}$$

or

$$n^{\mathrm{T}} = \frac{1}{\sqrt{2}} \begin{pmatrix} X_1 + iX_2 & X_3 + iX_4 & X_1 - iX_2 & X_3 - iX_4 \end{pmatrix},$$
(2.9)

whose images are both  $\mathbb{RP}^3 \subset \mathbb{CP}^3$  [13].

To provide a concrete example we remind the reader of the solution describing Dorey's dyonic magnon [10] on  $S^3$ ,

$$X^{1} + iX^{2} = e^{it/2} (\cos \frac{p}{2} + i \sin \frac{p}{2} \tanh \frac{u}{2}),$$
  

$$X^{3} + iX^{4} = e^{iv/2} \sin \frac{p}{2} \operatorname{sech} \frac{u}{2},$$
(2.10)

where

$$u = i(Z(\lambda_1) - Z(\bar{\lambda}_1)), \qquad v = Z(\lambda_1) + Z(\bar{\lambda}_1) - t$$
 (2.11)

in terms of

$$Z(\lambda) = \frac{x_+}{\lambda - 1} + \frac{x_-}{\lambda + 1}.$$
(2.12)

The resulting  $\mathbb{CP}^3$  solution involves putting two conjugate (and hence, oppositely charged) dyonic giant magnons together. The scaling of the world sheet coordinates (x, t)by 1/2 compared to [10] leaves the equation of motion (2.3) intact but is necessary in order to preserve the normalization of the Virasoro constraints (2.4). In the above we have used the parameterization  $\lambda_1 = re^{ip/2}$  of the spectral parameter, where p is the momentum of the magnon and r is related to its charge. In the limit  $r \to 1$  we recover the " $S^2 \times S^2$ " solution presented in [12].

More generally we can take any known giant magnon solution on  $S^3$  (such as the general *N*-magnon solution found in [20]) and embed it into the  $\mathbb{CP}^3$  model, thus obtaining a new classical string solution moving on  $\mathbb{R} \times \mathbb{CP}^3$  (again, it may be necessary to also scale the worldsheet coordinates to preserve the normalization given in (2.4)). Besides these 'trivial' solutions reviewed here, the  $\mathbb{CP}^3$  model admits more general solutions that can be obtained via the dressing method, to which we now turn our attention.

# 3 The dressing method for the $\mathbb{CP}^3$ coset

In order to apply the dressing method as outlined in [22] we first embed the  $\mathbb{CP}^3$  vector field n into an SU(4) principal chiral field g. This may be done by noting that

$$\mathbb{CP}^{3} = \{g \in \mathrm{SU}(4) : g\Omega g\Omega = 1\}, \quad \text{with } \Omega = \begin{pmatrix} +1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (3.1)$$

In order to understand this embedding, first observe that if g satisfies  $g^{\dagger}g = 1$  and  $g\Omega g\Omega = 1$ then the matrix  $1 + \Omega g$ 

$$P = \frac{1 + \Omega g}{2} \tag{3.2}$$

is a hermitian projection operator. Since det g = 1 it follows that det(2P - 1) = -1 so the rank of P must be either 1 or 3. In fact we can without loss of generality take P to have rank 1 since otherwise we could just replace  $P \rightarrow 1 - P$  throughout this analysis. Then we identify the vector n as the (unit-normalized) image of P.

Conversely, given a unit vector n we take

$$g = \Omega(2P - 1) \qquad \text{with } P = nn^{\dagger}, \tag{3.3}$$

which is easily seen to satisfy  $g\Omega g\Omega = 1$  and  $g \in SU(4)$ .

Under this embedding, the lagrangian (2.1) becomes proportional to

$$\mathcal{L} = \operatorname{Tr}\left[(g^{-1}\partial_{\mu}g)^{2}\right],\tag{3.4}$$

the equation of motion (2.3) becomes equivalent to the principal chiral model equation

$$\partial_{+}\partial_{-}g - \frac{1}{2}(\partial_{+}gg^{-1}\partial_{-}g + \partial_{-}gg^{-1}\partial_{+}g) = 0, \qquad (3.5)$$

while the Virasoro constraints (2.4) map into

$$\operatorname{Tr}\left[(g^{-1}\partial_{+}g)^{2}\right] = -2, \qquad \operatorname{Tr}\left[(g^{-1}\partial_{-}g)^{2}\right] = -2.$$
 (3.6)

Next we recall Theorem 4.2 of [22]. Given any solution g of the SU(4) principal chiral model which satisfies  $g\Omega g\Omega = 1$ , we first solve the auxiliary system

$$\partial_{+}\Psi = \frac{\partial_{+}gg^{-1}\Psi}{1-\lambda}, \qquad \partial_{-}\Psi = \frac{\partial_{-}gg^{-1}\Psi}{1+\lambda}$$
(3.7)

to find  $\Psi(\lambda)$  as a function of the auxiliary complex parameter  $\lambda$ , subject to the initial condition

$$\Psi(0) = g, \tag{3.8}$$

the SU(4) constraints

$$\det \Psi(0) = 1, \qquad [\Psi(\bar{\lambda})]^{\dagger} \Psi(\lambda) = 1, \tag{3.9}$$

as well as the coset constraint

$$\Psi(\lambda) = \Psi(0)\Omega\Psi(1/\lambda)\Omega. \tag{3.10}$$

With  $\Psi(\lambda)$  in hand a new dressed solution to the coset model may be constructed algebraically. The input to specify a new solution is an arbitrary complex parameter  $\lambda_1$ and an arbitrary complex four-vector e. In terms of this data the dressed solution is  $g' = \Psi'(0)$  where

$$\Psi'(\lambda) = \left[1 + \frac{Q_1}{\lambda - \lambda_1} + \frac{Q_2}{\lambda - 1/\lambda_1}\right]\Psi(\lambda)$$
(3.11)

in terms of two matrices  $Q_i = X_i F_i^{\dagger}$  specified by

$$F_1 = \Psi(\bar{\lambda}_1)e, \qquad F_2 = \Psi(0)\Omega\Psi(\bar{\lambda}_1)e \tag{3.12}$$

and the  $X_i$  are the solutions to

$$X_{1} \frac{F_{1}^{\dagger} F_{1}}{\lambda_{1} - \bar{\lambda}_{1}} + X_{2} \frac{F_{2}^{\dagger} F_{1}}{1/\lambda_{1} - \bar{\lambda}_{1}} = F_{1},$$

$$X_{1} \frac{F_{1}^{\dagger} F_{2}}{\lambda_{1} - 1/\bar{\lambda}_{1}} + X_{2} \frac{F_{2}^{\dagger} F_{2}}{1/\lambda_{1} - 1/\bar{\lambda}_{1}} = F_{2}.$$
(3.13)

# 4 Giant magnon solutions on $\mathbb{CP}^3$

In order to obtain new giant magnon solutions on  $\mathbb{CP}^3$  via the dressing method described in the previous paragraph we first choose as the vacuum

$$n^{\mathrm{T}} = \begin{pmatrix} \cos(t/2) & \sin(t/2) & 0 & 0 \end{pmatrix}.$$
 (4.1)

(a perhaps more obvious, but less useful, choice will be considered below). The scaling of the worldsheet time coordinate by 2 is necessary if we want our vacuum to satisfy the Virasoro constraints (2.4). Then the solution to the auxiliary system (3.7) that satisfies the initial condition and constraints is

$$\Psi(\lambda) = \begin{pmatrix} \cos Z(\lambda) & \sin Z(\lambda) & 0 & 0\\ -\sin Z(\lambda) & \cos Z(\lambda) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix},$$
(4.2)

where

$$Z(\lambda) = \frac{x_+}{\lambda - 1} + \frac{x_-}{\lambda + 1}.$$
(4.3)

Choosing (arbitrarily) the polarization vector

$$e^{\mathrm{T}} = \begin{pmatrix} 1 & 0 & i & 0 \end{pmatrix}. \tag{4.4}$$

we find the solution

$$n^{\mathrm{T}} = \frac{1}{\sqrt{R}} \begin{pmatrix} n^{1} & n^{2} & n^{3} & n^{4} \end{pmatrix},$$
 (4.5)

specified by

$$n^{1} = +2(1 - \lambda_{1}^{2})\bar{\lambda}_{1}\cos(t/2) + (1 - |\lambda_{1}|^{2})(\lambda_{1}\cos(t/2 - iu) + \bar{\lambda}_{1}\cos(t/2 + iu)) + (\lambda_{1} - \bar{\lambda}_{1})(\cos(t/2 - v) + |\lambda_{1}|^{2}\cos(t/2 + v)), n^{2} = -2(1 - \lambda_{1}^{2})\bar{\lambda}_{1}\sin(t/2) - (1 - |\lambda_{1}|^{2})(\bar{\lambda}_{1}\sin(t/2 + iu) + \lambda_{1}\sin(t/2 - iu)) - (\lambda_{1} - \bar{\lambda}_{1})(\sin(t/2 - v) + |\lambda_{1}|^{2}\sin(t/2 + v)), n^{3} = -2i(\lambda_{1} - \bar{\lambda}_{1})(1 - |\lambda_{1}|^{2})\cosh(u/2 + iv/2), n^{4} = 0.$$

$$(4.6)$$

In the above the normalization factor R is given by

$$R = \sum_{i=1}^{4} \bar{n}^{i} n^{i}$$
(4.7)

and

$$u = i(Z(\lambda_1) - Z(\bar{\lambda}_1)), \qquad v = Z(\lambda_1) + Z(\bar{\lambda}_1) - t.$$
(4.8)

The solution (4.6) is identical to the one presented recently in [14] (including, coincidentally, an almost identical choice of polarization vector (4.4)).

Upon parameterizing  $\lambda_1 = re^{ip/2}$  and evaluating the Noether charges using (2.2) we find that the solution (4.5) carries only a single nonzero charge J satisfying the dispersion relation (adapted to the conventions used in [24])

$$\Delta - \frac{1}{2}J = 2\sqrt{2\lambda}\frac{1+r^2}{2r}\left|\sin\frac{p}{2}\right|.$$
(4.9)

As usual for giant magnons, the charge J is itself infinite but the excitation energy  $\Delta - J$  of the magnon above the ground state (a pointlike string moving at the speed of light) is finite.

Remarkably the formula (4.9) is identical to the corresponding one for Dorey's dyonic giant magnon [10], but the solution (4.6) apparently carries only a single macroscopic charge and is hence not "dyonic" at all. The solution does reduce to the original Hofman-Maldacena magnon [9] with momentum p when the parameter r, whose physical interpretation at this point is mysterious, is taken to 1.

A second possible choice of vacuum is

$$n^{\mathrm{T}} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{it} & 1 & 0 & 0 \end{pmatrix},$$
 (4.10)

which differs from (4.1) by an SU(4) rotation which, importantly, does not commute with  $\Omega$ . In this case the solution to the linear system (3.7) is

$$\Psi(\lambda) = \frac{1}{\sqrt{1+\lambda}} \begin{pmatrix} \sqrt{\lambda} e^{+iZ(\lambda)} & +e^{+iZ(\lambda)} & 0 & 0\\ -e^{-iZ(\lambda)} & \sqrt{\lambda} e^{-iZ(\lambda)} & 0 & 0\\ 0 & 0 & \sqrt{1+\lambda} & 0\\ 0 & 0 & 0 & \sqrt{1+\lambda} \end{pmatrix}.$$
 (4.11)

If we choose as polarization vector

$$e^{\mathrm{T}} = \begin{pmatrix} 1 & i & 0 & 0 \end{pmatrix} \tag{4.12}$$

we find the solution

$$n^{1} = e^{+it/2} \left( (\lambda_{1} - \bar{\lambda}_{1}) |\lambda_{1}|^{2} e^{iv} - (\lambda_{1} - \bar{\lambda}_{1}) e^{-iv} - i(1 - |\lambda_{1}|^{2}) \lambda_{1} e^{u} - i(1 - |\lambda_{1}|^{2}) \bar{\lambda}_{1} e^{-u} \right),$$
  

$$n^{2} = e^{-it/2} \left( (\lambda_{1} - \bar{\lambda}_{1}) |\lambda_{1}|^{2} e^{-iv} - (\lambda_{1} - \bar{\lambda}_{1}) e^{iv} + i(1 - |\lambda_{1}|^{2}) \lambda_{1} e^{-u} + i(1 - |\lambda_{1}|^{2}) \bar{\lambda}_{1} e^{u} \right),$$
  

$$n^{3} = n^{4} = 0,$$
  
(4.13)

in terms u and v as before in (4.8). (This solution must of course also be normalized to unit length as in (4.5).) Remarkably this solution is identical to the bound state of two Hofman-Maldacena magnons on  $S^2$  found in (5.14) of [16], embedded into  $\mathbb{CP}^3$  via (2.5).

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### References

- [1] J. Bagger and N. Lambert, Modeling multiple M2's, Phys. Rev. D 75 (2007) 045020 [hep-th/0611108] [SPIRES];
  A. Gustavsson, Algebraic structures on parallel M2-branes, Nucl. Phys. B 811 (2009) 66 [arXiv:0709.1260] [SPIRES];
  J. Bagger and N. Lambert, Gauge symmetry and supersymmetry of multiple M2-branes, Phys. Rev. D 77 (2008) 065008 [arXiv:0711.0955] [SPIRES]; Comments on multiple M2-branes, JHEP 02 (2008) 105 [arXiv:0712.3738] [SPIRES];
  A. Gustavsson, Selfdual strings and loop space Nahm equations, JHEP 04 (2008) 083 [arXiv:0802.3456] [SPIRES].
- [2] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, N = 6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals, JHEP 10 (2008) 091
   [arXiv:0806.1218] [SPIRES].
- [3] M. Benna, I. Klebanov, T. Klose and M. Smedback, Superconformal Chern-Simons theories and AdS<sub>4</sub>/CFT<sub>3</sub> correspondence, JHEP 09 (2008) 072 [arXiv:0806.1519] [SPIRES].
- [4] G. Arutyunov and S. Frolov, Superstrings on AdS<sub>4</sub> × CP<sup>3</sup> as a coset σ-model, JHEP 09 (2008) 129 [arXiv:0806.4940] [SPIRES];
  B. Stefanski Jr., Green-Schwarz action for type IIA strings on AdS<sub>4</sub> × CP<sup>3</sup>, Nucl. Phys. B 808 (2009) 80 [arXiv:0806.4948] [SPIRES];
  P. Fré and P.A. Grassi, Pure spinor formalism for Osp(N|4) backgrounds, arXiv:0807.0044 [SPIRES];
  G. Bonelli, P.A. Grassi and H. Safaai, Exploring pure spinor string theory on AdS<sub>4</sub> × CP<sup>3</sup>, JHEP 10 (2008) 085 [arXiv:0808.1051] [SPIRES];
  J. Gomis, D. Sorokin and L. Wulff, The complete AdS<sub>4</sub> × CP(3) superspace for the type IIA superstring and D-branes, JHEP 03 (2009) 015 [arXiv:0811.1566] [SPIRES].
  [5] D. Gaiotto and X. Yin, Notes on superconformal Chern-Simons-matter theories,
- JHEP 08 (2007) 056 [arXiv:0704.3740] [SPIRES];
  J.A. Minahan and K. Zarembo, The Bethe ansatz for superconformal Chern-Simons, JHEP 09 (2008) 040 [arXiv:0806.3951] [SPIRES];
  D. Bak and S.-J. Rey, Integrable spin chain in superconformal Chern-Simons theory, JHEP 10 (2008) 053 [arXiv:0807.2063] [SPIRES];
  B.I. Zwiebel, Two-loop integrability of planar N = 6 superconformal Chern-Simons theory, arXiv:0901.0411 [SPIRES];
  J.A. Minahan, W. Schulgin and K. Zarembo, Two loop integrability for Chern-Simons theory theories with N = 6 supersymmetry, JHEP 03 (2009) 057 [arXiv:0901.1142] [SPIRES].
- [6] N. Gromov and P. Vieira, The all loop AdS<sub>4</sub>/CFT<sub>3</sub> Bethe ansatz, JHEP 01 (2009) 016 [arXiv:0807.0777] [SPIRES];
  C. Ahn and R.I. Nepomechie, N = 6 super Chern-Simons theory S-matrix and all-loop Bethe ansatz equations, JHEP 09 (2008) 010 [arXiv:0807.1924] [SPIRES].
- [7] T. McLoughlin and R. Roiban, Spinning strings at one-loop in AdS<sub>4</sub> × P<sup>3</sup>, JHEP 12 (2008) 101 [arXiv:0807.3965] [SPIRES];
  L.F. Alday, G. Arutyunov and D. Bykov, Semiclassical quantization of spinning strings in AdS<sub>4</sub> × CP<sup>3</sup>, JHEP 11 (2008) 089 [arXiv:0807.4400] [SPIRES];
  C. Krishnan, AdS<sub>4</sub>/CFT<sub>3</sub> at one loop, JHEP 09 (2008) 092 [arXiv:0807.4561] [SPIRES];
  T. McLoughlin, R. Roiban and A.A. Tseytlin, Quantum spinning strings in AdS<sub>4</sub> × CP<sup>3</sup>: testing the Bethe ansatz proposal, JHEP 11 (2008) 069 [arXiv:0809.4038] [SPIRES].

- [8] D. Berenstein and D. Trancanelli, Three-dimensional N = 6 SCFT's and their membrane dynamics, Phys. Rev. D 78 (2008) 106009 [arXiv:0808.2503] [SPIRES]; N. Drukker, J. Plefka and D. Young, Wilson loops in 3-dimensional N = 6 supersymmetric Chern-Simons theory and their string theory duals, JHEP 11 (2008) 019 [arXiv:0809.2787] [SPIRES]; B. Chen and J.-B. Wu, Supersymmetric Wilson loops in N = 6 super Chern-Simons-matter theory, arXiv:0809.2863 [SPIRES]; J. Kluson and K.L. Panigrahi, Defects and Wilson loops in 3D QFT from D-branes in  $AdS_4 \times CP^3$ , arXiv:0809.3355 [SPIRES]; S.-J. Rey, T. Suyama and S. Yamaguchi, Wilson loops in superconformal Chern-Simons theory and fundamental strings in Anti-de Sitter supergravity dual, JHEP 03 (2009) 127 [arXiv:0809.3786] [SPIRES]; C.-S. Chu and D. Giataganas, UV-divergences of Wilson loops for gauge/gravity duality, JHEP 12 (2008) 103 [arXiv:0810.5729] [SPIRES]; M. Fujita, BPS operators from the Wilson loop in the 3-dimensional supersymmetric Chern-Simons theory, arXiv:0902.2381 [SPIRES].
- [9] D.M. Hofman and J.M. Maldacena, Giant magnons, J. Phys. A 39 (2006) 13095 [hep-th/0604135] [SPIRES].
- [10] N. Dorey, Magnon bound states and the AdS/CFT correspondence, J. Phys. A 39 (2006) 13119 [hep-th/0604175] [SPIRES];
  H.-Y. Chen, N. Dorey and K. Okamura, Dyonic giant magnons, JHEP 09 (2006) 024 [hep-th/0605155] [SPIRES].
- [11] D. Gaiotto, S. Giombi and X. Yin, Spin chains in N = 6 superconformal *Chern-Simons-matter theory*, *JHEP* **04** (2009) 066 [arXiv:0806.4589] [SPIRES]; G. Grignani, T. Harmark, M. Orselli and G.W. Semenoff, Finite size giant magnons in the string dual of N = 6 superconformal Chern-Simons theory, JHEP 12 (2008) 008 [arXiv:0807.0205] [SPIRES]; C. Ahn and P. Bozhilov, Finite-size effects of Membranes on  $AdS_4 \times S^7$ , JHEP 08 (2008) 054 [arXiv:0807.0566] [SPIRES]; B. Chen and J.-B. Wu, Semi-classical strings in  $AdS_4 \times CP^3$ , JHEP 09 (2008) 096 [arXiv:0807.0802] [SPIRES]; D. Astolfi, V.G.M. Puletti, G. Grignani, T. Harmark and M. Orselli, Finite-size corrections in the SU(2) × SU(2) sector of type IIA string theory on  $AdS_4 \times CP^3$ , Nucl. Phys. B 810 (2009) 150 [arXiv:0807.1527] [SPIRES]; B.-H. Lee, K.L. Panigrahi and C. Park, Spiky strings on  $AdS_4 \times CP^3$ , JHEP 11 (2008) 066 [arXiv:0807.2559] [SPIRES]; I. Shenderovich, Giant magnons in  $AdS_4/CFT_3$ : dispersion, quantization and finite-size corrections, arXiv:0807.2861 [SPIRES]; R.C. Rashkov, A note on the reduction of the  $AdS_4 \times CP^3$  string  $\sigma$ -model, *Phys. Rev.* **D** 78 (2008) 106012 [arXiv:0808.3057] [SPIRES]; C. Ahn, P. Bozhilov and R.C. Rashkov, Neumann-Rosochatius integrable system for strings on  $AdS_4 \times CP^3$ , JHEP **09** (2008) 017 [arXiv:0807.3134] [SPIRES]; S. Ryang, Giant magnon and spike solutions with two spins in  $AdS_4 \times CP^3$ , JHEP 11 (2008) 084 [arXiv:0809.5106] [SPIRES]; D. Bombardelli and D. Fioravanti, Finite-size corrections of the  $CP^3$  giant magnons: the Lúscher terms, arXiv:0810.0704 [SPIRES]; T. Lukowski and O.O. Sax, Finite size giant magnons in the  $SU(2) \times SU(2)$  sector of

 $AdS_4 \times CP^3$ , JHEP 12 (2008) 073 [arXiv:0810.1246] [SPIRES];

C. Ahn and P. Bozhilov, Finite-size effect of the dyonic giant magnons in N = 6 super Chern-Simons theory, arXiv:0810.2079 [SPIRES]; B.-H. Lee and C. Park, Unbounded multi magnon and spike, arXiv:0812.2727 [SPIRES].

- [12] G. Grignani, T. Harmark and M. Orselli, The SU(2) × SU(2) sector in the string dual of N = 6 superconformal Chern-Simons theory, Nucl. Phys. B 810 (2009) 115
   [arXiv:0806.4959] [SPIRES].
- [13] M.C. Abbott and I. Aniceto, Giant magnons in  $AdS_4 \times CP^3$ : embeddings, charges and a hamiltonian, arXiv:0811.2423 [SPIRES].
- [14] T.J. Hollowood and J.L. Miramontes, Magnons, their solitonic avatars and the pohlmeyer reduction, JHEP 04 (2009) 060 [arXiv:0902.2405] [SPIRES].
- [15] V.E. Zakharov and A.V. Mikhailov, Relativistically invariant two-dimensional models in field theory integrable by the inverse problem technique. (In russian), Sov. Phys. JETP 47 (1978) 1017 [Zh. Eksp. Teor. Fiz. 74 (1978) 1953] [SPIRES]; On the integrability of classical spinor models in two-dimensional space-time, Commun. Math. Phys. 74 (1980) 21 [SPIRES].
- [16] M. Spradlin and A. Volovich, Dressing the giant magnon, JHEP 10 (2006) 012 [hep-th/0607009] [SPIRES].
- [17] C. Kalousios, M. Spradlin and A. Volovich, Dressing the giant magnon. II, JHEP 03 (2007) 020 [hep-th/0611033] [SPIRES].
- [18] R. Ishizeki, M. Kruczenski, M. Spradlin and A. Volovich, Scattering of single spikes, JHEP 02 (2008) 009 [arXiv:0710.2300] [SPIRES].
- [19] A. Jevicki, C. Kalousios, M. Spradlin and A. Volovich, Dressing the giant gluon, JHEP 12 (2007) 047 [arXiv:0708.0818] [SPIRES];
  A. Jevicki, K. Jin, C. Kalousios and A. Volovich, Generating AdS string solutions, JHEP 03 (2008) 032 [arXiv:0712.1193] [SPIRES];
  A. Jevicki and K. Jin, Solitons and AdS string solutions, Int. J. Mod. Phys. A 23 (2008) 2289 [arXiv:0804.0412] [SPIRES].
- [20] C. Kalousios, G. Papathanasiou and A. Volovich, Exact solutions for N-magnon scattering, JHEP 08 (2008) 095 [arXiv:0806.2466] [SPIRES].
- [21] I. Aniceto and A. Jevicki, N-body dynamics of giant magnons in  $R \times S^2$ , arXiv:0810.4548 [SPIRES].
- [22] J.P. Harnad, Y. Saint Aubin and S. Shnider, Backlund transformations for nonlinear σ-models with values in riemannian symmetric spaces, Commun. Math. Phys. 92 (1984) 329 [SPIRES].
- [23] R. Suzuki, Giant magnons on CP<sup>3</sup> by dressing method, JHEP 05 (2009) 079
   [arXiv:0902.3368] [SPIRES].
- [24] M.C. Abbott, I. Aniceto and O.O. Sax, Dyonic giant magnons in CP<sup>3</sup>: strings and curves at finite J, arXiv:0903.3365 [SPIRES].
- [25] T.J. Hollowood and J.L. Miramontes, A new and elementary CP<sup>n</sup> dyonic magnon, arXiv:0905.2534 [SPIRES].